

## Advanced Signal Processing Algorithms for Sound and Vibration

– Beyond the FFT

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This presentation will introduce several advanced signal processing algorithms for sound and vibration that go beyond the FFT. These advanced algorithms can solve some sound and vibration challenges that FFT-based algorithms cannot solve. This presentation will introduce the background of these algorithms and their application examples, such as bearing fault detection, dashboard motor testing and speaker testing.

## Agenda

- Advanced Signal Processing Algorithms
  - Time-Frequency Analysis
  - Quefrency and Cepstrum
  - Wavelet Analysis
  - AR Modeling
- Application Examples
  - Bearing fault detection, dashboard motor testing, speaker testing, ...

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Many sound and vibration applications have adopted signal processing. FFT-based signal processing algorithms, for example, power spectrum and total harmonic distortion (THD) measurements are the most widely used. However, FFT-based signal processing algorithms might not help in some applications.

This presentation will introduce several advanced signal processing algorithms beyond FFT. These advanced algorithms can solve some sound and vibration challenges that FFT-based algorithms cannot solve. This presentation will introduce the background of these algorithms and their application examples, such as bearing fault detection, dashboard motor testing and speaker testing.

## Sound and Vibration Signals

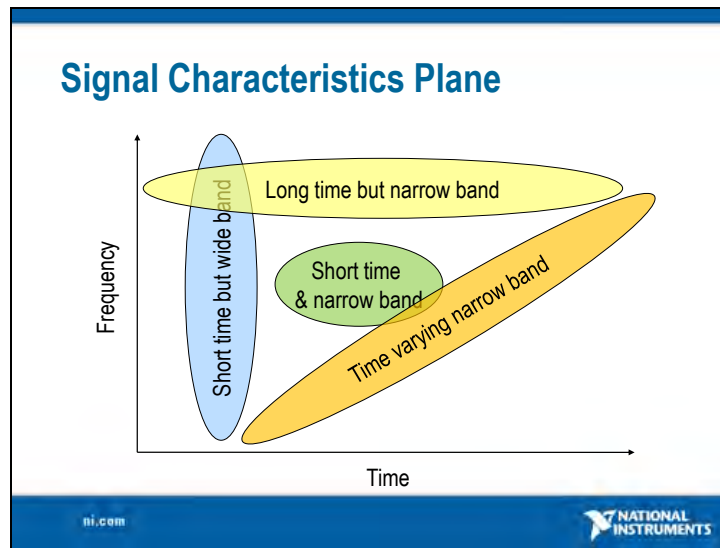
- Can indicate the condition or quality of machines and structures
  - Cooling fans with faulty bearings produce louder noise
- You can analyze sound and vibration signals to
  - Optimize a design
  - Ensure production quality
  - Monitor machine or structure conditions

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Sound and vibration signals can indicate the condition or quality of machines and devices. Many machines and devices generate noise or vibration when they operate. You can analyze sound and vibration signals when you design and manufacture a product. You can also analyze sound and vibration signals to monitor the conditions of critical machines in their working state as well as structural health.

For example, when a cooling fan spins, it generates noise. High-quality cooling fans produce lower levels of noise. Defective cooling fans (for example, ones with a bearing defect or a broken blade) produce higher levels of noise.



Before you select the right algorithm, you need to understand the characteristics of the signal first.

If we look at the signal characteristics in the time-frequency plane, we can understand the signal characteristics better.

- Some features have a long time duration but narrow bandwidth, for example, rub & buzz noise.
- Some features have a short time duration but wide bandwidth, for example, spikes and breakdown points.
- Some features have a short time duration and narrow bandwidth, for example, decayed resonance.
- Some features might have a time-varying bandwidth, for example, the imbalance bearing generating noise dependent on RPM.

You might use different signal processing algorithms for different types of signal characteristics in the time-frequency plane.

## Signal Processing Algorithms Overview

- Time Domain
- Frequency Domain
- Time-Frequency Domain
- Quefrequency Domain (Cepstrum)
- Wavelet
- Model-Based

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There are many signal processing algorithms that you can use to extract signal features. Based on the independent variable in the algorithms, they can be classified into time domain, frequency domain, time-frequency domain, quefrequency domain (cepstrum), wavelet and AR model-based.









We'll focus on time-frequency analysis, cepstrum, wavelet, and model-based algorithms in this presentation.

### How to Select the Right Algorithms

	Frequency Analysis	Order Analysis	Time-Frequency Analysis	Quefreny Analysis	Wavelet Analysis	Model Based
			?			

There many algorithms that you can use. So the question is how to select a correct algorithms. We'll look at each algorithm and see their fit in this table.

### Select the Right Algorithms

	Frequency Analysis	Order Analysis	Time-Frequency Analysis	Wavelet Analysis	Model Based
					
					
					
					
					

Frequency analysis is inherently suitable for analyzing signals with narrow band or harmonic frequency components that do not change over time.

Order analysis is suitable for analyzing time-varying signals that are dependent on the RPM of rotational machines.

We'll present the fundamentals of other analysis algorithms and see where they fit.

## Limitations of the FFT

- No information about how frequencies evolve over time
- Not suitable for analyzing impulsive signals

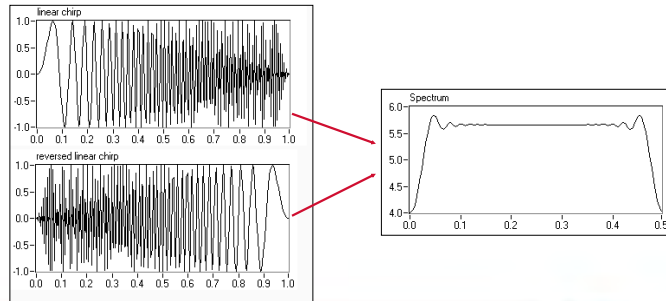
Before we talk about time-frequency analysis, let's look at frequency analysis first.

Frequency analysis is the most commonly used analysis method and is very useful in many applications. The FFT is the basic operation in frequency analysis. Frequency analysis results, such as power spectrum and THD, contain only frequency information of the signal. These results do not contain time information. Frequency analyses are useful for analyzing stationary signals whose frequency components do not change over time.

For short-time duration signal components, frequency analysis might not be useful because the short-time duration signal components might have low power and be drowned in the spectrum of noise.

## Power Spectrum

- A power spectrum does not contain time information



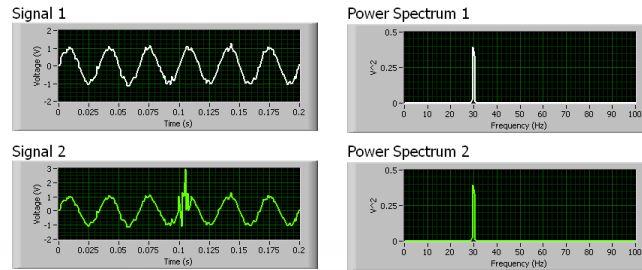
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The simplest example is to compute a linear chirp and its time-reversed version. While frequencies of one chirp signal *increase* with time (top left), frequencies of the other chirp signal *decrease* with time (bottom left). Although the frequency behavior of the two signals is obviously different, their frequency spectra (right) computed by the FFT are identical! As a matter of fact, there is an infinite number of completely different signals that can produce the same spectrum!

## Transients

- It is difficult to detect presence of transients in a signal by its power spectrum



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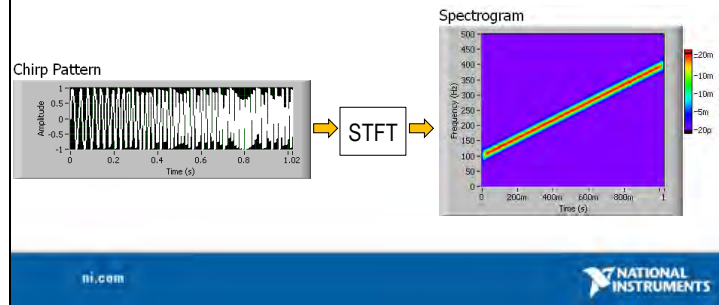
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Transients are sudden events that last for a short time. They usually have low energy and wide frequency band. When they are transformed into frequency domain, their energy will spread over a wide range in the frequency domain. Since they have low energy, you might not be able to recognize their existence in the frequency domain.

# Time-Frequency Analysis

## Time-Frequency Analysis

- The short-time Fourier transform (STFT) is the most popular time-frequency analysis algorithm

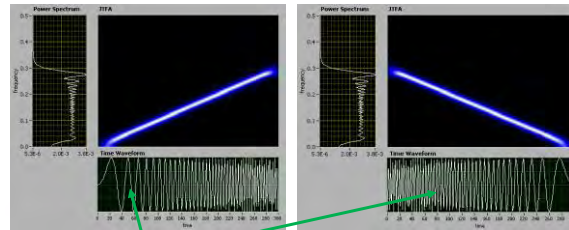


The time-frequency analysis results are usually given in a spectrogram. A spectrogram shows how the energy of a signal is distributed in the time-frequency domain. A spectrogram is an intensity graph with two independent variables: time and frequency. The x-axis is time, and the y-axis is frequency. The color intensity shows the power of the signal at the corresponding time and frequency.

A chirp pattern is a signal whose frequency linearly increases over time. From the power spectrum of the chirp pattern, you can only see the frequency components of the signal, which are from 100Hz to 400Hz. However, the spectrogram shows how the frequency of the chirp pattern changes over time. You can see that the frequency increases from 100Hz to 400Hz in one second.

## Advantages of Time-Frequency Analysis

- Time-frequency representation shows how frequency components of a signal evolve over time



Reversed in time domain

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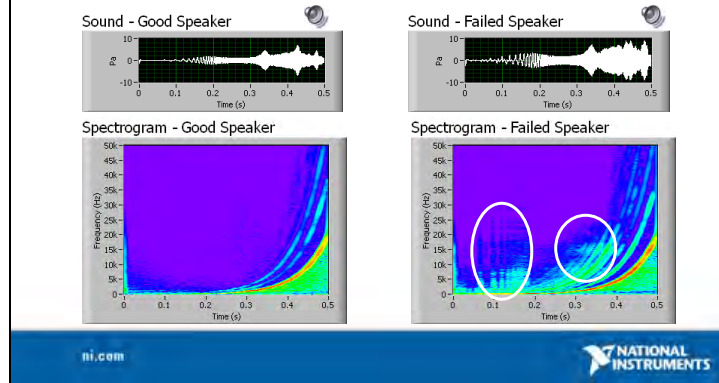
Time-frequency analysis represents a signal in time-frequency domain. These results reveal how the frequency components of a signal change over time. Time-frequency analysis is suitable for analyzing time-varying signals.

Some signals might have a narrow frequency band and last for a short time duration. These signals can have a good concentration in the time-frequency domain. Noise signals usually are distributed in the entire time-frequency domain. So the time-frequency representation might be able to improve local signal-to-noise ratio in the time-frequency domain. That means you might recognize the existence of a signal that might not be recognized in other domain. You'll see an example in the following slides.

Time-frequency representation also can help you understand characteristics of a signal and select the right signal processing algorithm to process the signal.

## Application Example: Speaker Test










- Speakers play a log chirp for quality test



Time-frequency analysis can be used in production testing. Here is an example in speaker production test. In production testing, speakers play a log chirp. Operators listen to the speaker and judge the quality of the speakers.

A log chirp is a time-varying signal whose frequency changes from 10Hz to 20KHz. You can use time-frequency analysis algorithms to analyze the sound generated by a speaker to judge the quality of the speaker.

The spectrogram of a good speaker is very “clean”. You can see the good speaker generates the expected frequency components (log-chirp) except there are harmonics, which are acceptable if the harmonics are not that high. Conversely, the spectrogram of the failed speaker contains many abnormal components.

Select the Right Algorithms						
	Frequency Analysis	Order Analysis	Time-Frequency Analysis	Quefrequency Analysis	Wavelet Analysis	Model Based
						
						
						
						
						

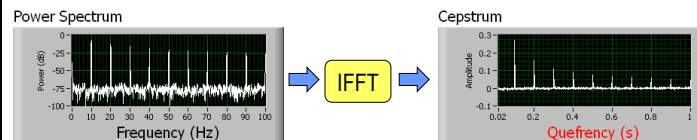
This table is a rule of thumb in selecting the right algorithm based on the time-frequency characteristics. Note that these are guidelines only.

- If the signal is a narrow-band signal that lasts for a long time, use frequency analysis.
- If the signal contains harmonics and lasts for a long time, use quefrequency analysis.
- If the signal is a wide-band signal and lasts for a very short time, use wavelet analysis or AR modeling.
- If the signal is time-varying, use time-frequency analysis.
- If the signal is a narrow-band signal and lasts for a short time, use wavelet analysis.

# Quefreny Analysis

## Cepstrum and Quefrency

- Cepstrum is the spectrum of a decibel spectrum
- Quefrency is the independent variable of cepstrum



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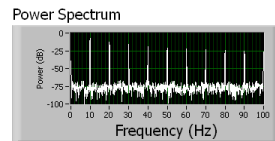
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“Cepstrum” was derived from “spectrum” by reversing the first four letters of “spectrum”. A cepstrum is the inverse FFT of the log of a spectrum. The independent variable of power spectrum is frequency. Correspondingly, the independent variable of cepstrum is called quefrency. The name of quefrency was derived from “frequency” by reversing the first three letters and second three letters of “frequency”.

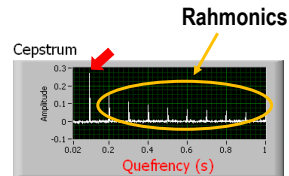
Quefrency is a measure of time. But it’s not in the sense of time domain. A spectrum reveals the periodicity of a time domain signal, while a cepstrum reveals the periodicity of a spectrum. You can consider cepstrum as the spectrum of a spectrum.

## Cepstrum Property

- The cepstrum reveals the periodicity of a spectrum
  - A peak in the cepstrum corresponds to harmonics in power spectrum



10Hz harmonics

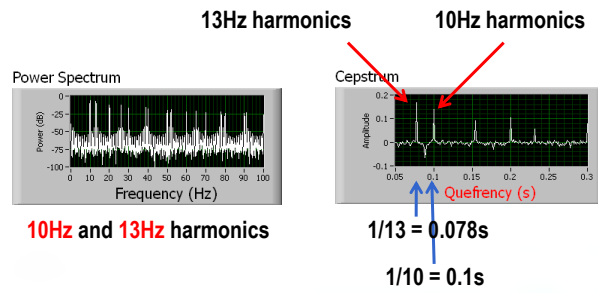


A peak at 0.1s quefrency

Another property of a cepstrum is that it can reveal the periodicity of a spectrum. Spectra are the easiest tools to use to understand the periodicity of a signal. So a cepstrum is also called the spectrum of a spectrum.

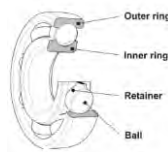
Harmonics are very common in spectra. Harmonics are periodic components in a spectrum. So we can use a cepstrum to detect whether there are harmonics in a spectrum. One of the applications is bearing fault detection.

## Cepstrum Property Cont.



## Application Example: Bearing Fault Detection

- Use a cepstrum to detect a **bearing fault**



Characteristic frequency for an **outer** ring fault of a bearing

$$f_{outer} = \frac{N_B f}{2} \left( 1 - \frac{D_B}{D_C} \cos(\alpha) \right)$$

Characteristic frequency for an **inner** ring fault of a bearing

$$f_{inner} = \frac{N_B f}{2} \left( 1 + \frac{D_B}{D_C} \cos(\alpha) \right)$$

$N_B$ : Number of balls     $D_B$ : Ball diameter     $\alpha$ : Ball contact angle

$f$ : Rotation frequency     $D_C$ : Retainer diameter

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With the property in the previous slide, we can use cepstrum to detect faults in a bearing.

A ball bearing is mainly composed of an outer ring, an inner ring, and several balls. If there are faults in the outer ring or inner ring, the vibration signal will become larger in some frequency components. We call these frequency components *characteristic frequencies*. The equations in the slide show the characteristic frequency for outer ring faults and inner ring faults. The characteristic frequencies are related to the number of balls, the RPM, and geometry parameters of the bearing components.

## Bearing Fault Detection Example

- Geometry parameters of the bearings under test are:

$$N_B = 7 \quad f = 30\text{Hz} \quad D_C = 70\text{mm} \quad D_B = 10\text{mm} \quad \alpha = 0$$

- Characteristic frequencies of the bearings are:

$$\text{Outer ring fault - } f_{\text{outer}} = \frac{N_B f}{2} \left( 1 - \frac{D_B}{D_C} \cos(\alpha) \right) = 3.0 f = 90\text{Hz}$$

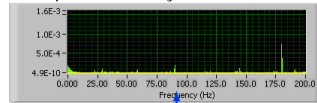
$$\text{Inner ring fault - } f_{\text{inner}} = \frac{N_B f}{2} \left( 1 + \frac{D_B}{D_C} \cos(\alpha) \right) = 4.0 f = 120\text{Hz}$$

The numbers in this slide shows the parameters of a real bearing and its characteristic frequencies of outer ring faults and inner ring faults.

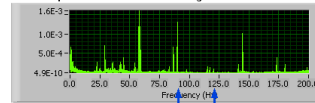
## Power Spectrum of Bearing Signals

- **90Hz** peak in the power spectrum of an **outer** ring fault signal
- **120Hz** peak in the power spectrum of an **inner** ring fault signal

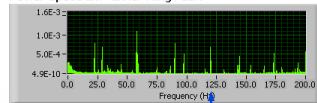
Power Spectrum - Outer Ring Fault



Power Spectrum - Normal Bearing



Power Spectrum - Inner Ring Fault



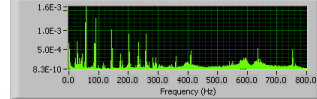
A **90Hz** peak is also obvious in the power spectrum of a normal bearing

In this example, the power spectrum of the bearing with a fault in its **outer** ring has a peak at **90Hz**, and the power spectrum of the bearing with fault in its **inner** ring has a peak at **120Hz**, which are as expected. However, we can also find an obvious 90Hz peak in the power spectrum of a good bearing. This means peaks in the characteristic frequencies might not be good enough to differentiate between good and faulty bearings.

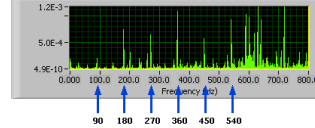
## Harmonics of Bearing Signals

- Use harmonics to detect bearing faults
  - The **outer** ring fault signal has harmonics of **90Hz**
  - The **inner** ring fault signal has harmonics of **120Hz**

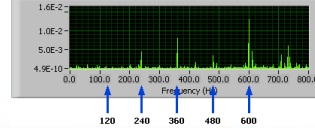
Power Spectrum - Normal Bearing



Power Spectrum - Outer Ring Fault



Power Spectrum - Inner Ring Fault



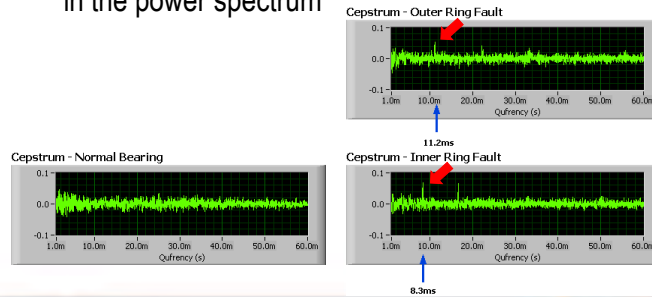
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If you look at the power spectrum globally, you can find the harmonics of the characteristic frequencies are obvious. The harmonics are not obvious in the power spectrum of a good bearing. So using harmonics is a more reliable way to detect bearing faults.

## Cepstrum of Bearing Signals











- A peak in the cepstrum means harmonics exist in the power spectrum



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A cepstrum is good way to detect harmonics in a spectrum. The cepstrum of the bearing with a fault in its outer ring has an obvious peak at about 11.2ms which corresponds to harmonics of about 90Hz. The cepstrum of the bearing with fault in its inner ring has an obvious peak at about 8.3ms which corresponds to harmonics of about 120Hz. The cepstrum of the good bearing does not have obvious peaks.

Select the Right Algorithms						
	Frequency Analysis	Order Analysis	Time-Frequency Analysis	Quefrency Analysis	Wavelet Analysis	Model Based
						
						
						
						
						

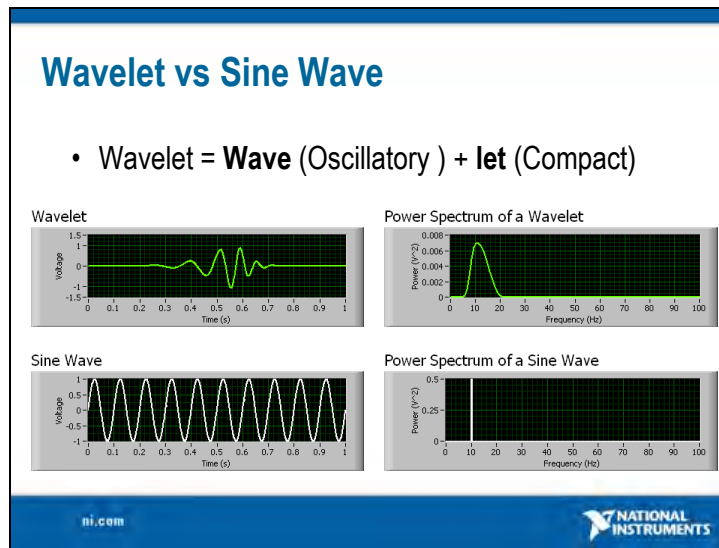
### Highlights of Cepstrum Analysis

- Good for deconvolution
  - Applications include RPM detection and echo detection
- Good for detecting harmonics
  - Applications include bearing fault detection and gearbox fault detection

# Wavelet Analysis

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Wavelets are defined as signals with two properties: *admissibility* and *regularity*.

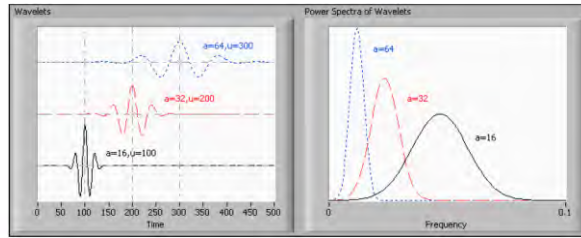
- *Admissibility* means that wavelets must have a band-pass like spectrum. Admissibility also means that wavelets must have a zero average in the time domain. A zero average implies that wavelets must be oscillatory.
- *Regularity* states that the wavelets have some smoothness and concentration in both the time and frequency domains. So wavelets are oscillatory and compact signals.

As a comparison, sine waves oscillate along the time axis forever without any decay, which means they are not compact. So, sine waves do not have any concentration in the time domain. On the other hand, sine waves have extreme concentration in frequency domain, which is a delta. Sine waves have maximum resolution in frequency domain but no resolution in time domain. For example, if I shift a sine wave with its periods, you cannot realize I have shifted it at all.

Wavelets have limited bandwidth in the frequency domain and compact bandwidth in the time domain. So, wavelets have a good concentration and resolution trade-off between time and frequency domain.

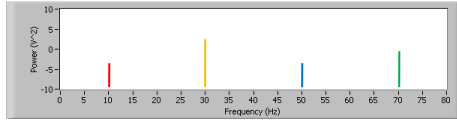
## Multi-Resolution

- A higher scale wavelet has larger time duration but lower frequency and smaller bandwidth



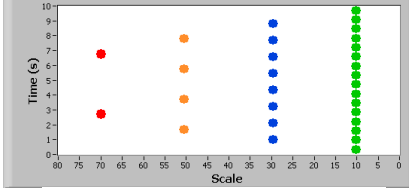
## Wavelet Transform: Look at the FFT First

Power Spectrum



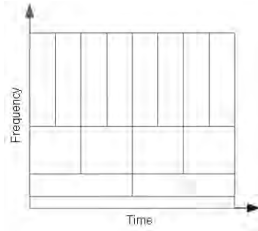
# Wavelet Transform

Scalogram

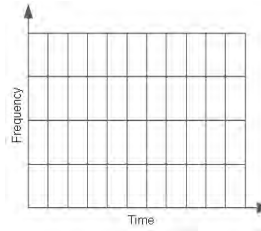


## Wavelet Transform vs STFT

- A wavelet transform has adaptive time-frequency resolution



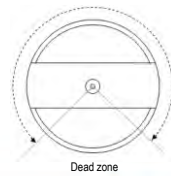
Time-frequency resolution  
of Wavelet Transform



Time-frequency resolution  
of STFT

## Application Example: Dashboard Motor Production Test

- A dashboard motor is a stepper motor that has an angle constraint
  - Oil pressure, tachometers, and speedometers use dashboard motors



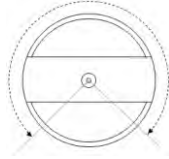
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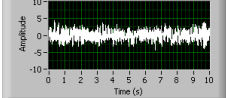
Here is an example that uses wavelet analysis in the production test of a dashboard motor. A dashboard motor is a stepper motor that has an angle constraint. Instead of rotating in 360 degree, there is a dead zone. It can only rotate between two angles. Oil meters, tachometers, and speedometers on a instrument panel all use this kind of motor.

## Dashboard Motor Faults

- There are two kinds of faults
  - Fault 1 – Knock at turning angles
  - Fault 2 – Rub noise

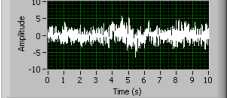


Vibration Signal of a Good Motor



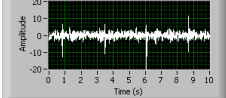
Good Motor

Vibration Signal of Faulty Motor 1




Knocks

Vibration Signal of Faulty Motor 2



Larger Knocks  
and Rub

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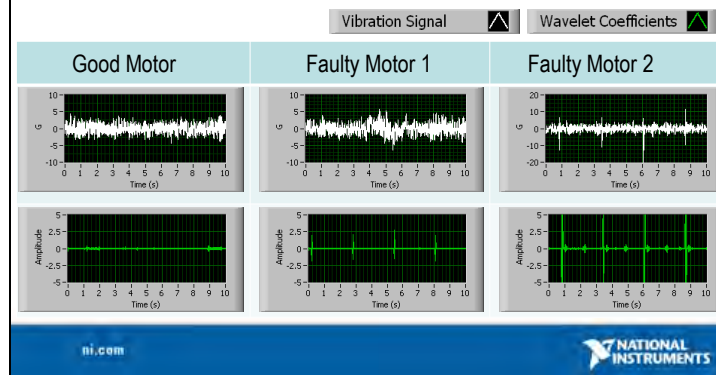
This production test is mainly designed to detect two kinds of faults in a dashboard motor:

- Knocks at turning angles
- Rub when rotating

Listening to the signal of faulty motor 1, you can hear “Da-Da” sounds, which are knocks at the turning angles. The faulty motor 2 has more obvious knocks. In addition, the faulty motor 2 has rub noise, which sounds like “Zee-Zee”. As a comparison, there are no Da-Da or Zee-Zee sounds in the signal of a good motor.

## Use Wavelet Transform to Detect Motor Faults

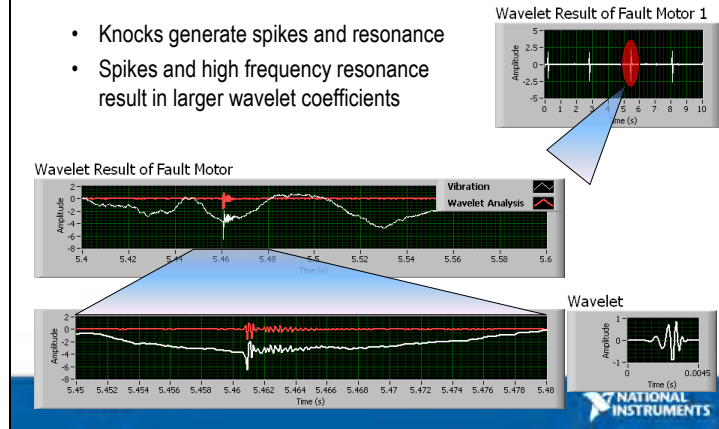
- Larger wavelet coefficients indicate existence of faults



You can use a wavelet transform to detect the knocks and rubs in the signal. From the wavelet result plots, you can see the knocks result in greater wavelet coefficients. Rubs also result in relatively greater wavelet coefficients. The wavelet coefficients for a good motor are smaller when compared to those of the faulty motor.

## Why do Wavelets Work?

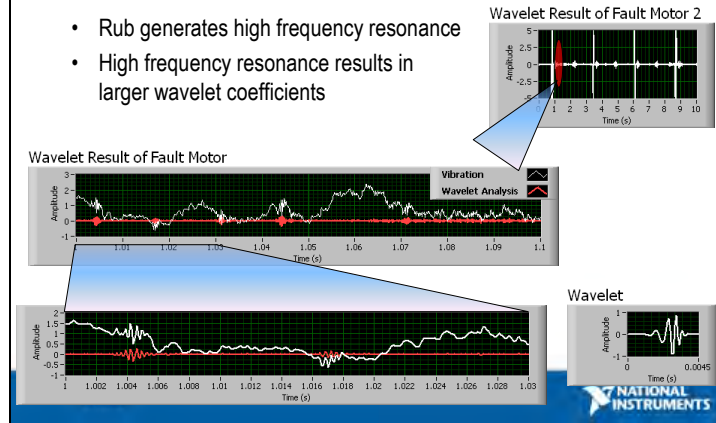
- Knocks generate spikes and resonance
- Spikes and high frequency resonance result in larger wavelet coefficients



Why do the knocks and rubs result in greater wavelet coefficients? If you zoom in on the signal to see the details when knocks occur, you will see knocks are spikes and resonance in the signal. Spikes and resonance are relatively high-frequency components. If you use a wavelet that has similar bandwidth with these signal components, you'll get large wavelet coefficients. You also can understand it as a pattern matching problem. If a signal segment matches a wavelet, that segment gets a high score (wavelet coefficient).






## Why do Wavelets Work? (Cont.)

- Rub generates high frequency resonance
- High frequency resonance results in larger wavelet coefficients



Similar to knock, rub generates high frequency resonance which results in larger wavelet coefficients.

### Select the Right Algorithms

	Frequency Analysis	Order Analysis	Time-Frequency Analysis	Quefreny Analysis	Wavelet Analysis	Model Based
	👉					
	👉			👉		
		👉	👉			
					👉	
					👉	

### Highlights of Wavelet Analysis

Good for transient signal detection

E.g., Spike, Edge, Break Point, Peaks/Valleys.

Multi-resolution Analysis

Easy to find signal events in different scale (E.g., both wide peaks and narrow peaks)

# Model-Based Analysis

## Auto-Regressive (AR) Modeling

- A sample in a time series can be considered as the linear combination of past samples plus error

$$x(n) = \sum_{k=1}^M a_k x(n-k) + e(n)$$

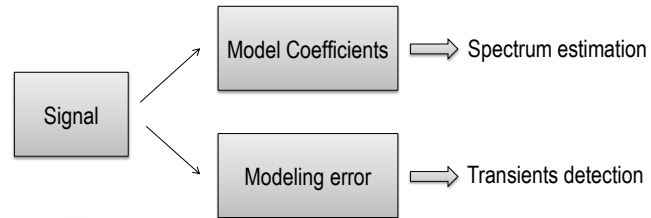
Deterministic part  
(Model Coefficients)

Stochastic part  
(Modeling error)

You can consider a signal as the deterministic part plus the stochastic part. The deterministic part can be represented by a linear model while the stochastic part is random and cannot be represented by a linear model.

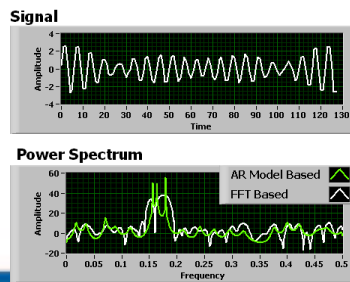
Auto-Regressive (AR) modeling is a commonly-used model. An AR model represents any sample in a time series as the linear combination of the past samples in the same time series. The white noise in the time series cannot be picked up by the linear combination. The modeling error  $e(n)$  corresponds to the noise that cannot be picked up by the linear combination.

## AR Modeling Applications



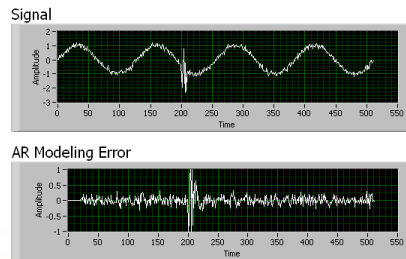
## Power Spectrum Estimation

- The AR model spectrum has higher resolution than the FFT based spectrum



## AR Modeling of Non-stationary Signals

- The AR modeling errors indicate the existence of transients in a signal.



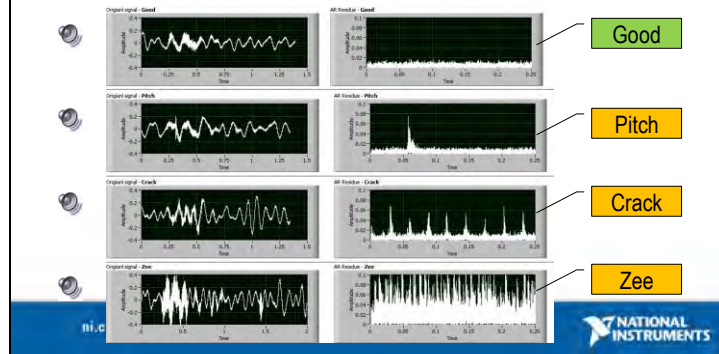
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If there are transients in a signal, there might be transients in the modeling error. As shown in the example in this slide, there is a spike in the sine wave. Because the majority of the signal is the sine wave, AR can represent the sine wave well. But the AR model cannot pick up the spike and the white noise. So the spike will be part of the AR modeling error.

## Application Example: Hard Disk Drive Production Test

- AR modeling errors indicate different types of HDD faults.



Here is an example that uses AR modeling to detect hard disk drive faults. You can listen to the sounds of the HDDs. From the sounds of the faulty HDDs, you can hear obvious transients (pitch, crack, and Zee). The AR modeling error indicates the transients clearly.

### Application Example: Engine Knock Detection

- Optimized ignition timing results in a higher degree of engine efficiency
  - Earlier ignition results in a lower engine temperature and reduced efficiency.
  - Late ignition might result in auto-ignition and cause engine knocks, which are shock waves on the cylinder.
- Engine knocks are transient events and can be detected by the AR modeling error.

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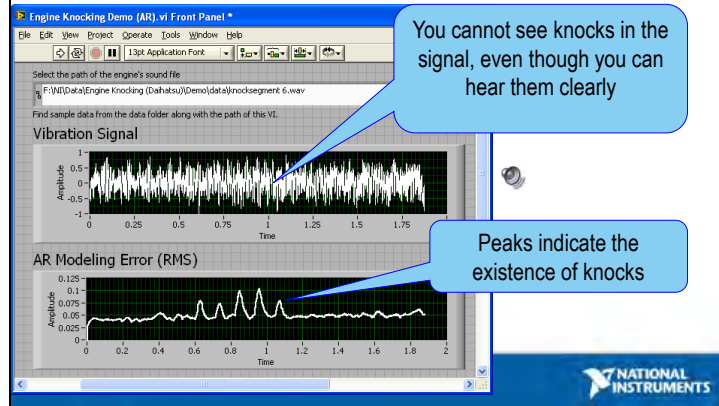
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Another application example of AR modeling is engine knock detection.

In a gasoline engine, spark plugs ignite to burn the mixture of air and fuel. The timing of ignition is very important and will affect the efficiency and fuel economy of the engine. If the ignition is optimized, the mixture burns smoothly from the point of ignition to the cylinder walls. If the ignition is late, the mixture might be automatically ignited when the temperature of the mixture exceeds a critical level. This auto-ignition produces a shock wave that generates a rapid increase in cylinder pressure. When auto-ignition occurs, the engine might make a knocking noise.

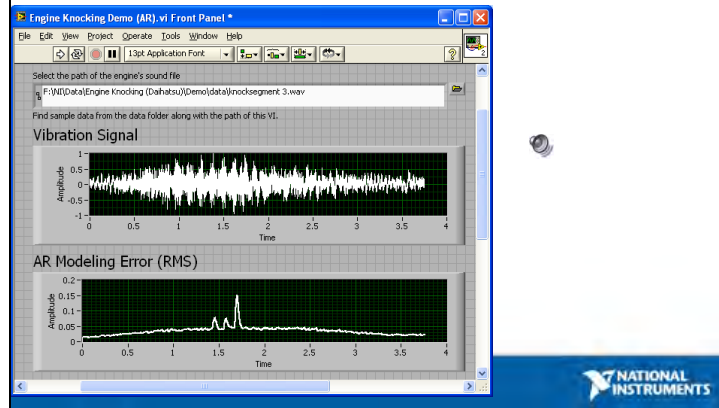
From the signal aspect, the signal samples are very different from others when engine knocks occur. Knocks produce transients in the signal. So it is possible to detect engine knocks by using AR modeling.

## Engine Knocking Detection - Sample 1 Constant Speed



If you listen to the vibration signal of the engine, you can hear five knocks clearly. However, you cannot see it from the vibration signal. If you apply an AR model for the signal, you can clearly see these five peaks in the AR modeling error. These peaks correspond to the knocks.

## Engine Knocking Detection - Sample 2 Run-up and Run-down



This is a similar example with different data samples. The difference is that this engine was run up and down. If the RPM changes smoothly and not that fast, you can still apply the AR modeling method to detect the engine knocks.

## Highlights of AR Modeling

- Good mathematical description of stationary signal.
- The AR modeling error indicates transients in the signal

### Select the Right Algorithms

	Frequency Analysis	Order Analysis	Time-Frequency Analysis	Quefrequency Analysis	Wavelet Analysis	Model Based
	👉					👉
	👉			👉		
		👉	👉			
					👉	👉
					👉	

This table is a rule of thumb in selecting the right algorithm based on the time-frequency characteristics. Note that these are guidelines only.

- If the signal is a narrow-band signal that lasts for a long time, use frequency analysis.
- If the signal contains harmonics and lasts for a long time, use quefrequency analysis.
- If the signal is a wide-band signal and lasts for a very short time, use wavelet analysis or AR modeling.
- If the signal is time-varying, use time-frequency analysis.
- If the signal is a narrow-band signal and lasts for a short time, use wavelet analysis.